Theory Introduction to PC decomposition.

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# Theory

Introduction to PC decomposition.

### The PC decomposition

Assume that X is a multivariate random variable, with probability distribution function f. Consider the nonlinear function Y=g(X), where g is a square integrable nonlinear function.

The polynomial chaos decomposition of g is



where P is the number of multivariate chaos polynomials and  is the k-th multivariate chaos polynomial. These multivariate polynomials are constructed from tensorisation of univariate orthogonal polynomials.

The coefficients of the PC decomposition satisfy the equation:



The denominator of the previous equation can be computed in advance, since the associated integrals only depend on the chaos polynomials, which have known integrals. Therefore, only the numerator has to be computed. There are two different methods to do this: integration or regression.

### Orhogonal polynomials

In this section, we present the orthogonal polynomials which are used to create the model. For each distribution associated to the random variable, we use a different type of orthogonal polynomial. The chosen family of polynomials has the property that they are orthogonal with respect to the weight associated with the probability distribution function of the variable.

|  |  |
| --- | --- |
| Distribution | Polynomial |
| "Uniforme" | Legendre |
| "LogUniforme" | Legendre |
| "Normale" | Hermite |
| "LogNormale" | Hermite |
| "Exponentielle" | Laguerre |

Let us denote by  a family of univariate orthogonal polynomials. The associated multivariate chaos polynomial is



where  is the i-th multi-indice associated with the polynomial .

### Integration and regression

The two methods to compute the PC decomposition in this toolbox are integration, based on quadrature multidimensionnal rules, and regression, based on the solution of a linear least squares problem.

The integration method uses a multidimensionnal quadrature based on the formula



where n is the number of integration points,  are the integration weights and  are the integration points.

An integration rule, such as the "Quadrature" sampling, for example, defines the number of integration points, the weights and the integration points.

In the regression method, the coefficients of the PC decomposition are computed by solving a linear least squares problem. The coefficients  minimize



Hence, the type of sampling used to create the polynomial chaos decomposition must be consistent with the method used to compute the coefficients of the decomposition. The name argument of the setrandvar\_buildsample function must be consistent with the method argument of the polychaos\_computeexp function. These two functions are generally combined, as in the following fragment of script.

setrandvar\_buildsample(srvx,name,np)  
 [...]  
 polychaos\_computeexp(pc,srvx,method)

The next table presents the available combinations.

|  |  |  |
| --- | --- | --- |
| Sample name | "Regression" | "Integration" |
| "MonteCarlo" | OK |  |
| "Lhs" | OK |  |
| "QmcSobol" | OK |  |
| "Quadrature" |  | OK |
| "Petras" |  | OK |
| "SmolyakGauss" |  | OK |
| "SmolyakFejer" |  | OK |
| "SmolyakTrapeze" |  | OK |
| "SmolyakClenshawCurtis" |  | OK |

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