SCILAB WORKSHOP

Optimization in SCILAB

SCILAB TEAM
17th, October 2019

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Advanced post-processing

Airfoil shape optimization

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Quick introduction to Numerical Optimization

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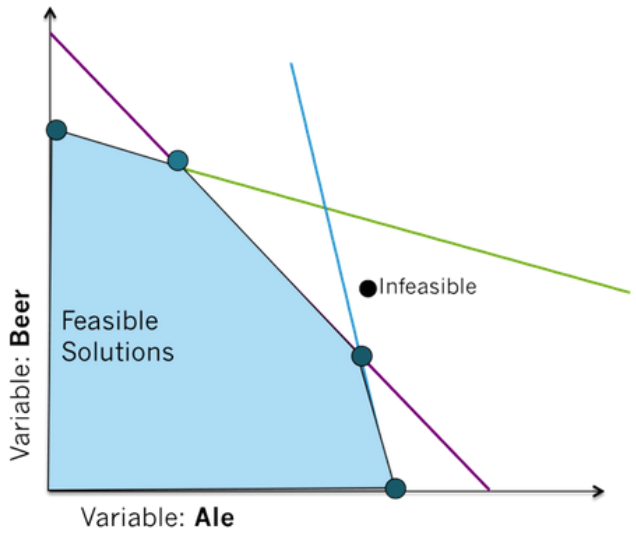
Optimization problem

$$\boxed{\max_{x \in X} f(x)}$$

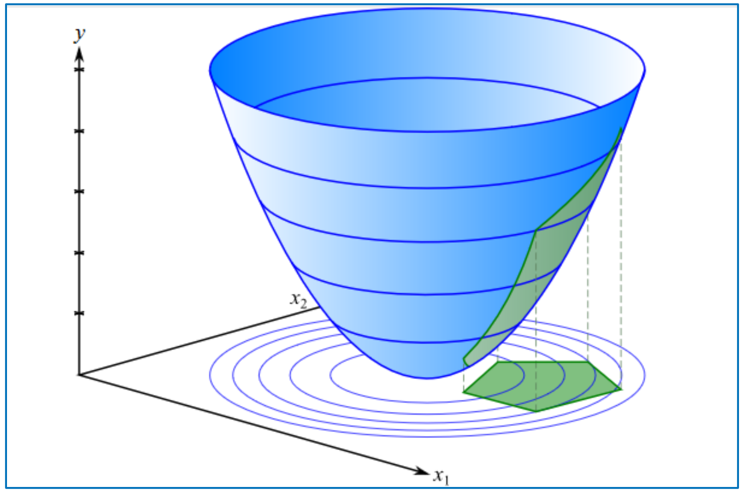


3 Steps:

1/ Identification of the decision variables: x

2/ Set-up of constraints associated to the variables: X

3/ Definition of a cost/objective function: f



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Two formulations, One problem

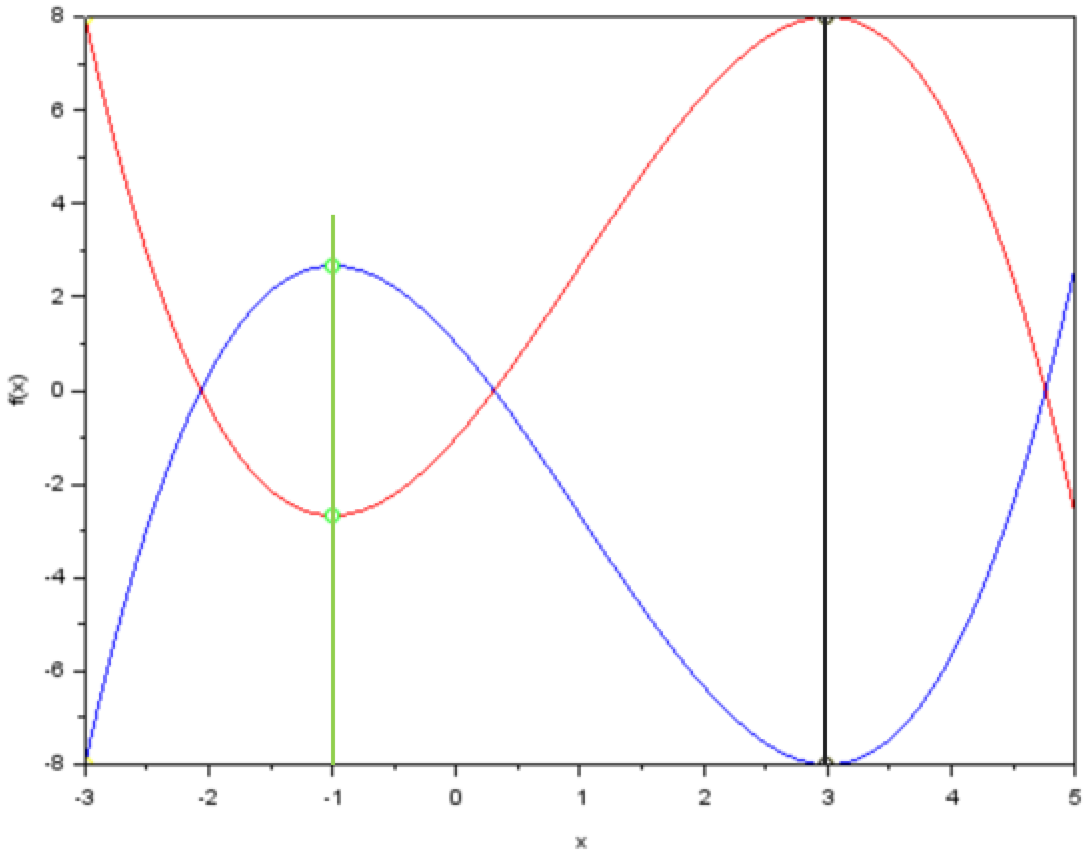
Minimize or Maximize a function?

Solving the problem

$$\min_{x \in X} f(x)$$

Corresponds to solve the problem

$$\max_{x \in X} -f(x)$$



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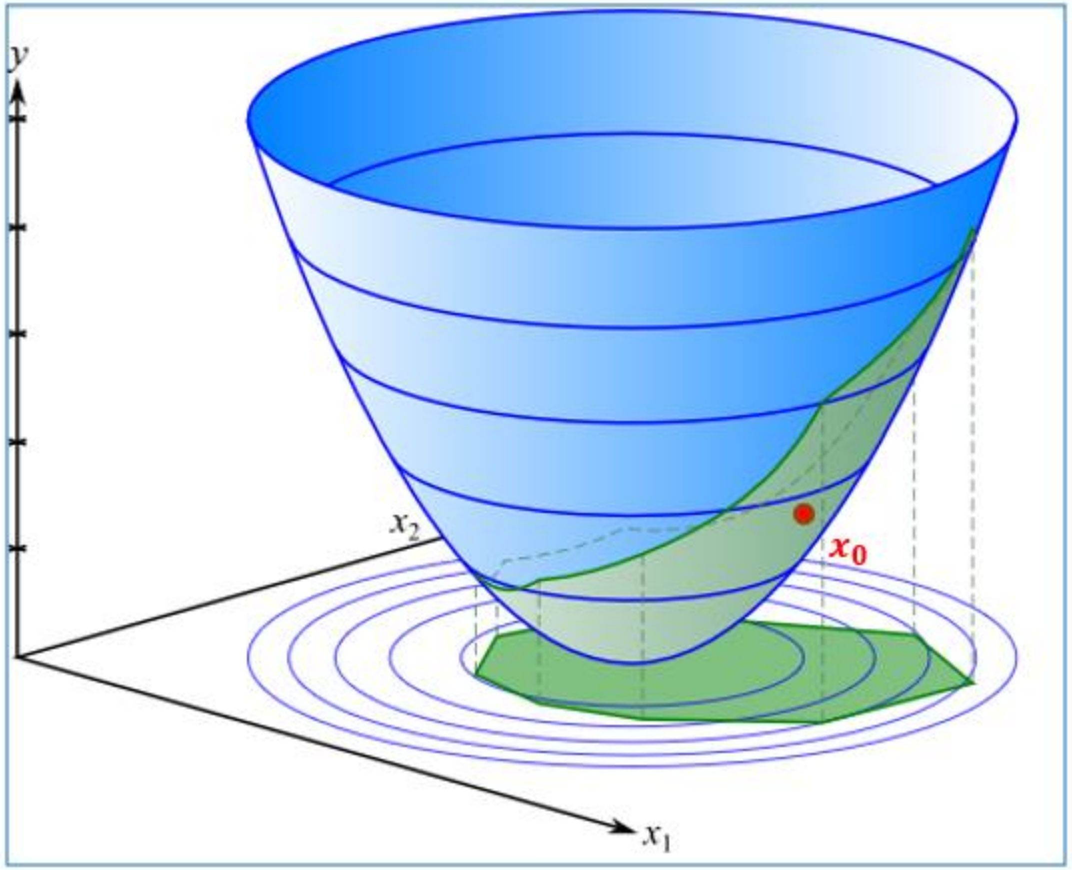
Numerical optimization



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Classical set-up parameters for optimization solvers

1/ Initial approximation– Influence convergence
2/ Number of iterations– Recursive process
3/ Convergence speed
4/ Stopping criteria

- Maximum number of iterations, …

- Cost function value under a given threshold, …

$$\|f(x_n)\| < \varepsilon_1$$

- Difference between two consecutive approximations under a given threshold, …

$$\|x_{n+1} - x_n\| < \varepsilon_2$$

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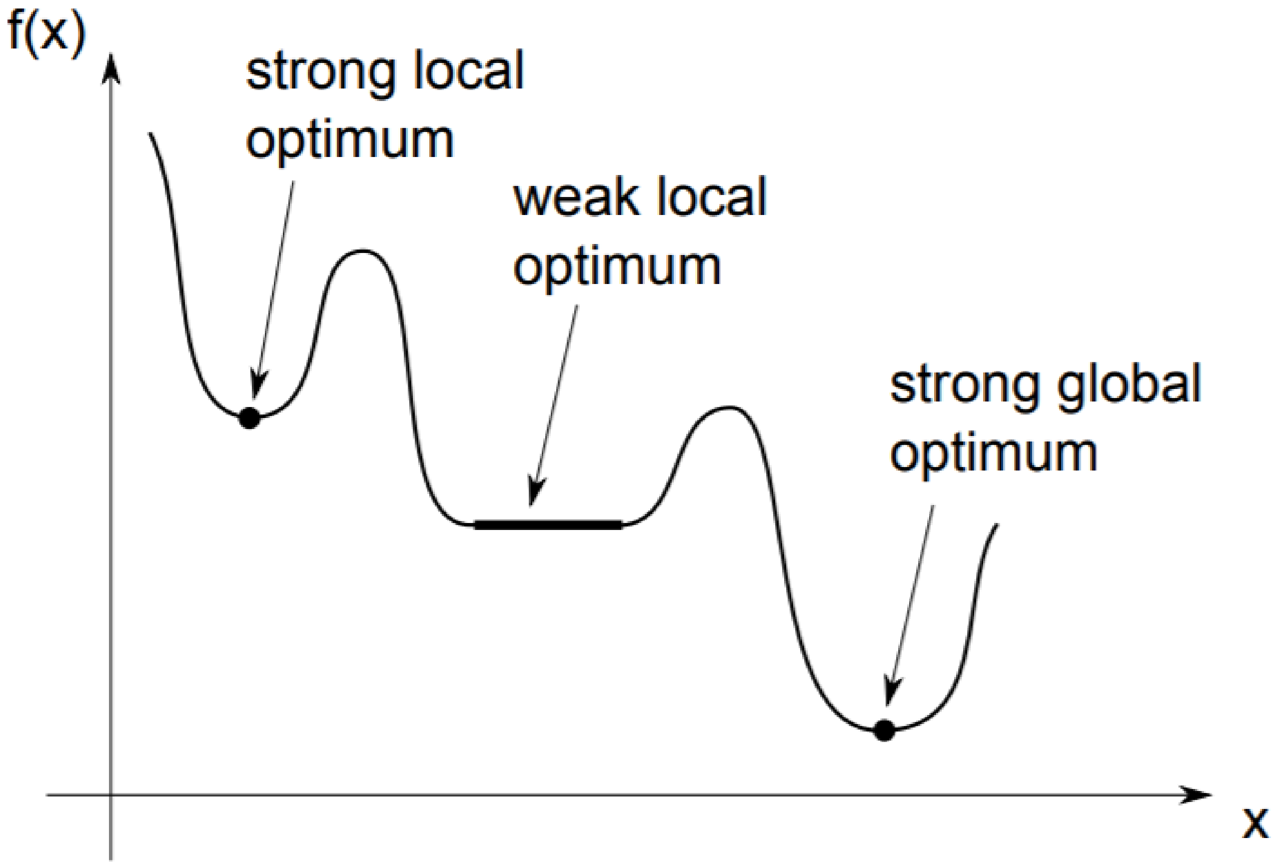
WARNING : Different types of extrema



<u>General remarks</u>

- Local extrema are easy to find!
- Some algorithm are performing local search only!

Take care on initial approximation!

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Maximum search methods

How to make design space exploration?

Direct search

Search for an optimal position using cost function evaluations, without any computation of the function exact or approximate derivatives.

Interior Points method, simplex

Derivatives computation

Search for an optimal position using cost function derivatives (Jacobian for steepest ascent direction, Hessian for critical point type) exact or approximate computation.

Gradient method, Newton method

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Optimization problem typology

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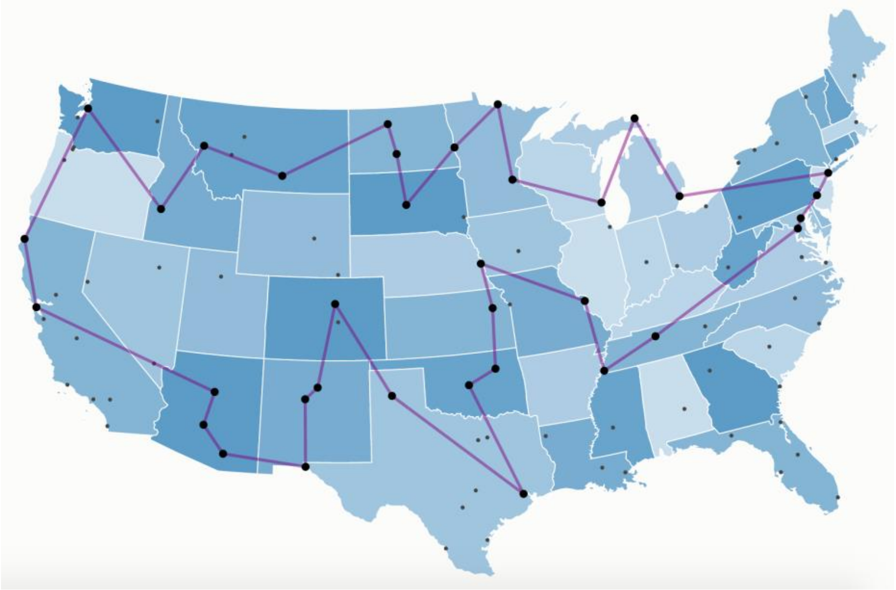
Continuous vs Discrete

DISCRETE (Combinatorial Analysis)

Variables are taking values in a finite set or states (IN, ON/OFF, …).

Classical examples: Travelling salesman problem which consists in finding the shortest path binding a given set of cities.



Travelling salesman problem

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Mono vs Multi objective

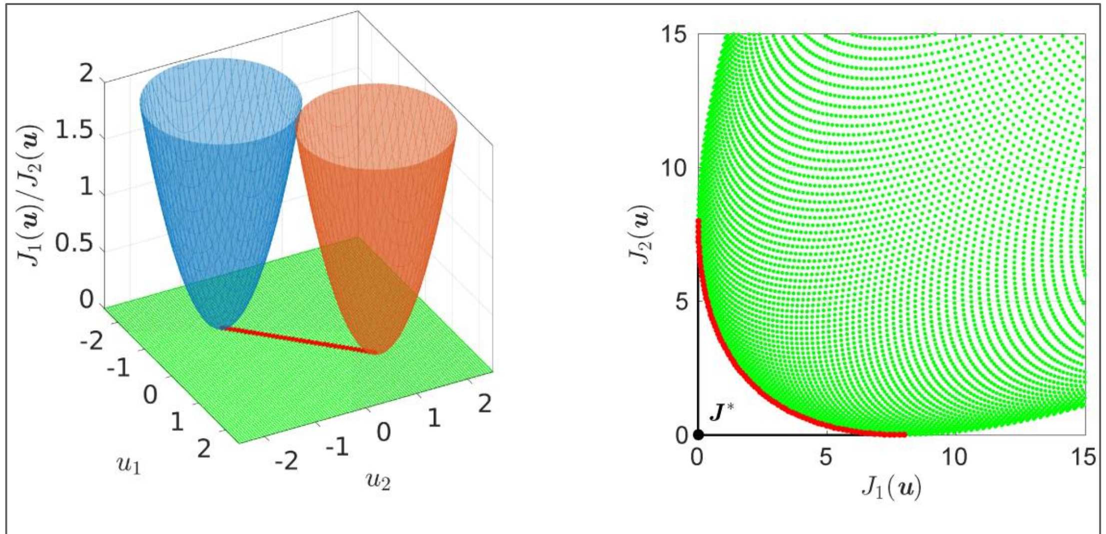
MULTI OBJECTIVE

$$\min_{x \in X}(f_1(x), f_2(x), \dots, f_k(x))$$

Several cost, potentially conflicting, functions minimization.

Found solution is therefore not unique. The complete set of solutions is called *Pareto* Front.

Choice is about trade-off



(a) (b)

An example would be to ensure maximum efficiency, and minimum investment at the same time.

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Constrained vs Non Constrained

CONSTRAINTS

Constraints are referring to the limit of the design space we impose

- Linear

$$Cx = b$$

- Non Linear

$$x^T C x = b$$

- Equalities

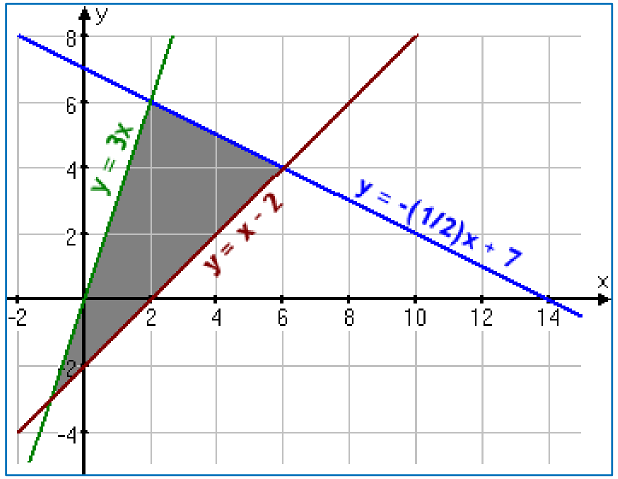
$$Cx = b$$

- Inequalities

$$Cx \geq b$$

- Lower and Upper bounds

$$a_1 \leq x \leq a_2$$



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Problem size

Let n be the number of design variables

- $n < 10$: Small problem (s)
- $10 < n < 100$: Medium problem (m)
- $100 < n < 1000$: Large problem (l)

Solver speed and memory needs are depending on the problem size

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Optimization solvers in SCILAB

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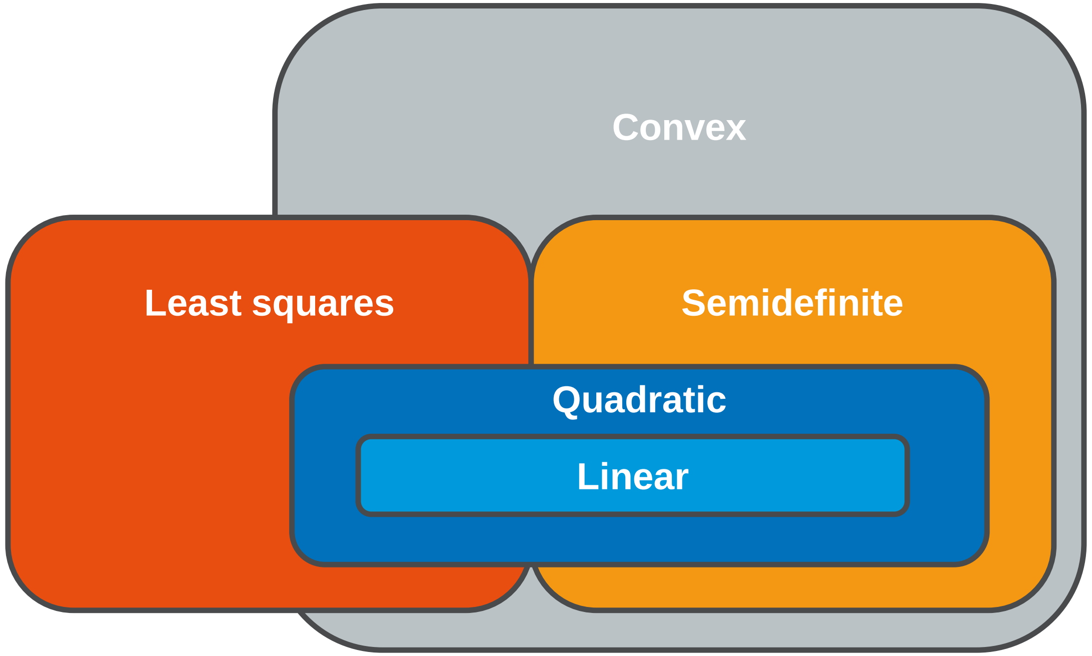
How to choose SCILAB optimization solver



Objective	Constraints [Bnds, Eq, Ineq]	Size	Gradient	Solver
Linear	Y Y Y	m		karmarkar
Quadratic	Y Y Y	L		qpsolve
SemiDef	N Y Y	L		lmisolver
SemiDef	Y N Y	L		semidef
N.L.S.	N N N	L	optional	Leastsq / lsqrsolve
Non-linear	Y N N	L	Y	optim
Non-linear	N N N	s		fminsearch

Objective	Single Objective	Multi Objective	Use
Continuous/ Differentiable	Fminsearch		
Non Smooth	optim		
Discrete	Optim_sa, optim_ga	Optim_moga Optim_nsga2	

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Linear optimization

Linear problem

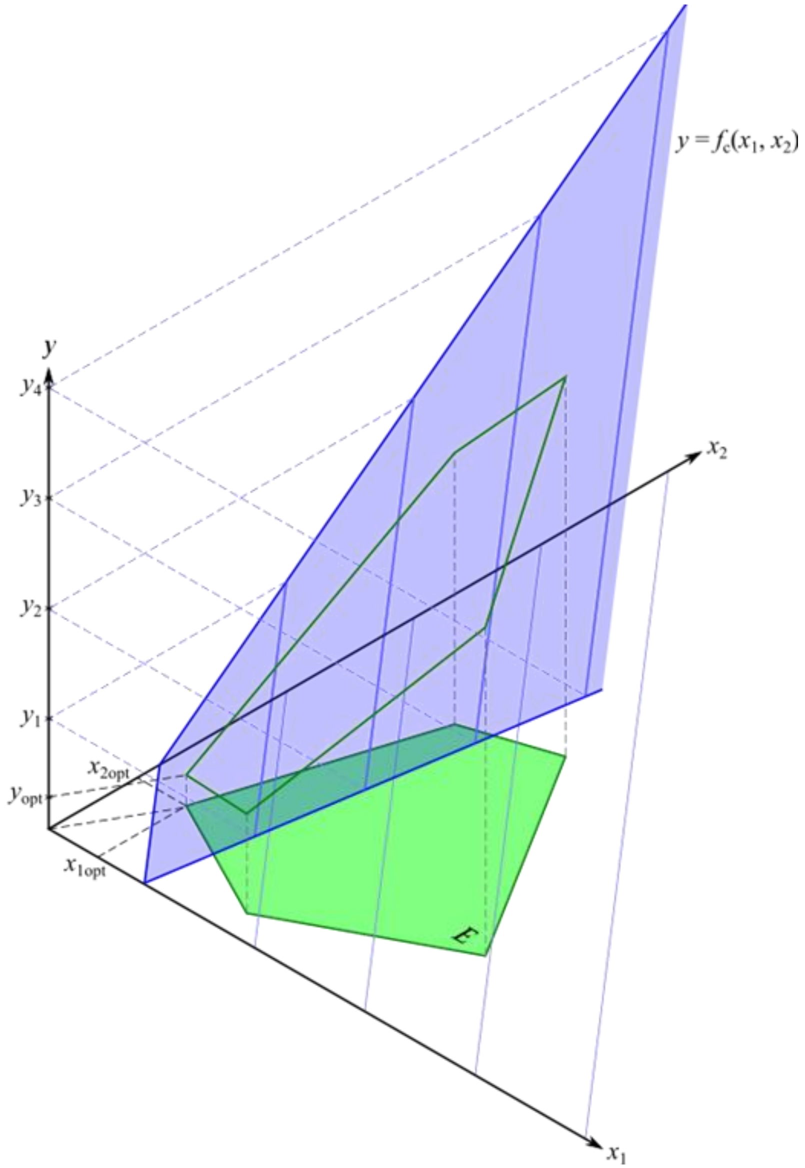
$$\min_{x} f(x) = c^T x$$

Linear constraints

- Equalities
- Inequalities

$$C_1^T x = b_1$$

$$C_2^T x \geq b_2$$



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Linear optimization - karmarkar

Interior Points Method

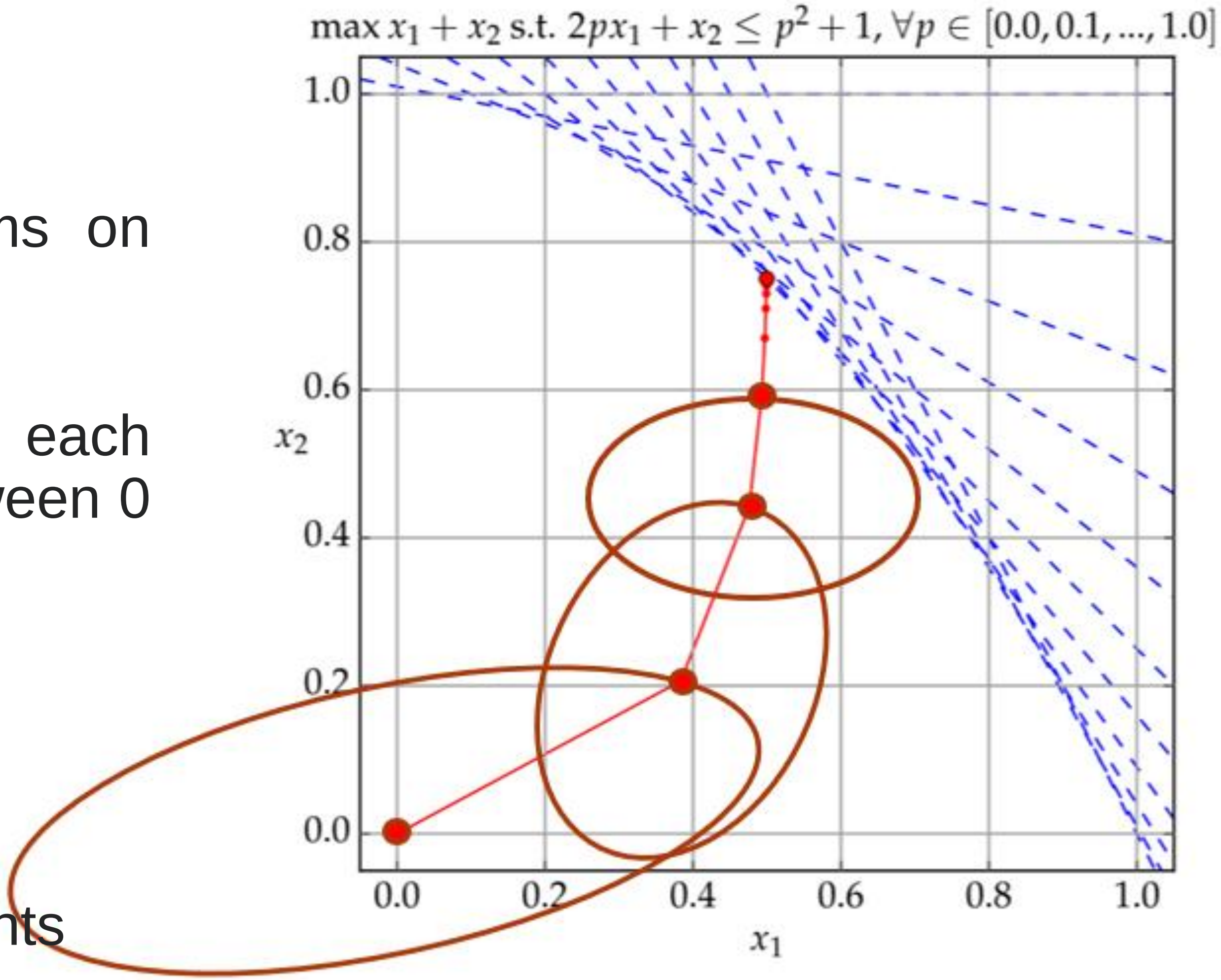
« Affine Scaling »

Solve suite of optimization problems on ellipsoid.

Ellipsoids size are decreasing at each iteration by a given scale factor (between 0 and 1).

Caracteristic:

- Polynomial time $O(n^m)$
- Direct search
- (In)Equalities and bounds constraints



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Quadratic optimization

Quadratic problem

$$\min_{x} f(x) = \frac{1}{2} x^T Q x + c^T x$$

With Q la « hessian » and c « gradient » of a virtual energy function

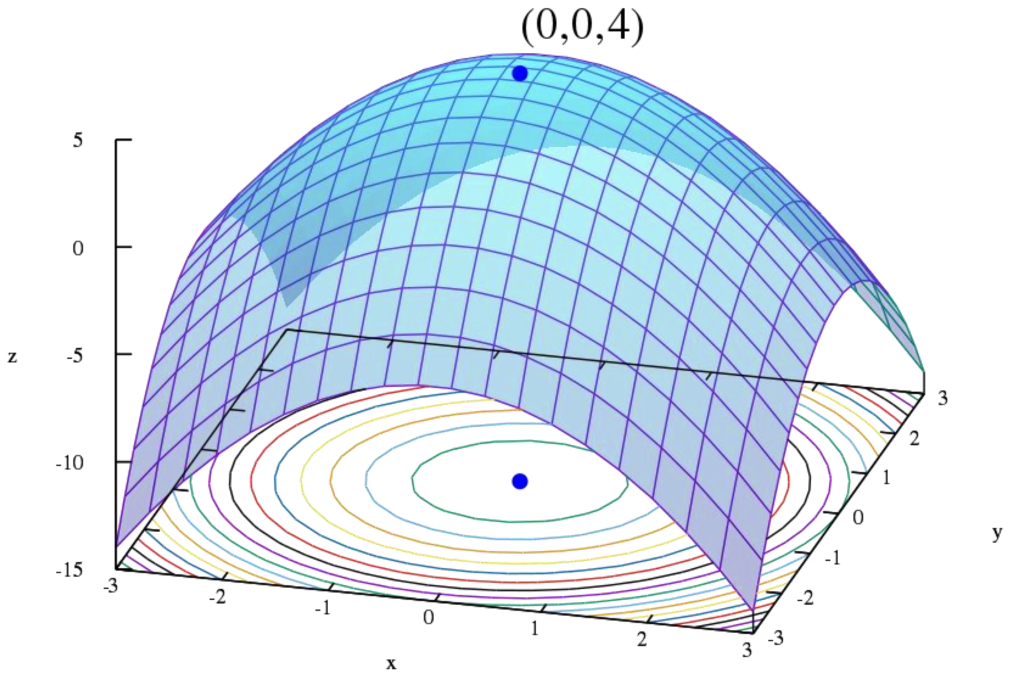
Linear constraints

- Equalities

$$C_1^T x = b_1$$

- Inequalities

$$C_2^T x \geq b_2$$



Concave shape, convex problem

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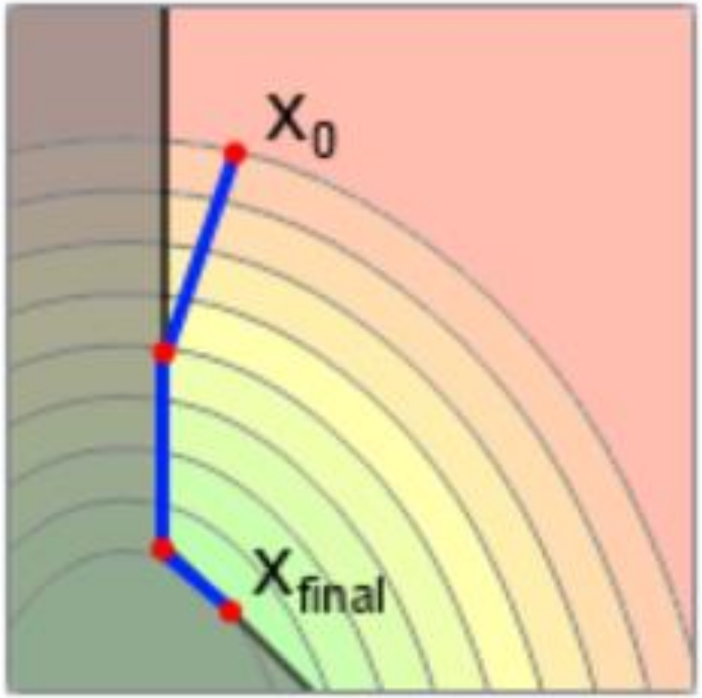
Quadratic optimization - qpsolve

Goldfarb-Idnani solver

« Feasible active set method »

Find non-constrained global optimum, then add non-respected constraints one after the other (activation)



Caracteristics :

- Strictly convex problems
- (In)Equalities and bounds constraints
- Polynomial time $O(n^m)$
- Direct search

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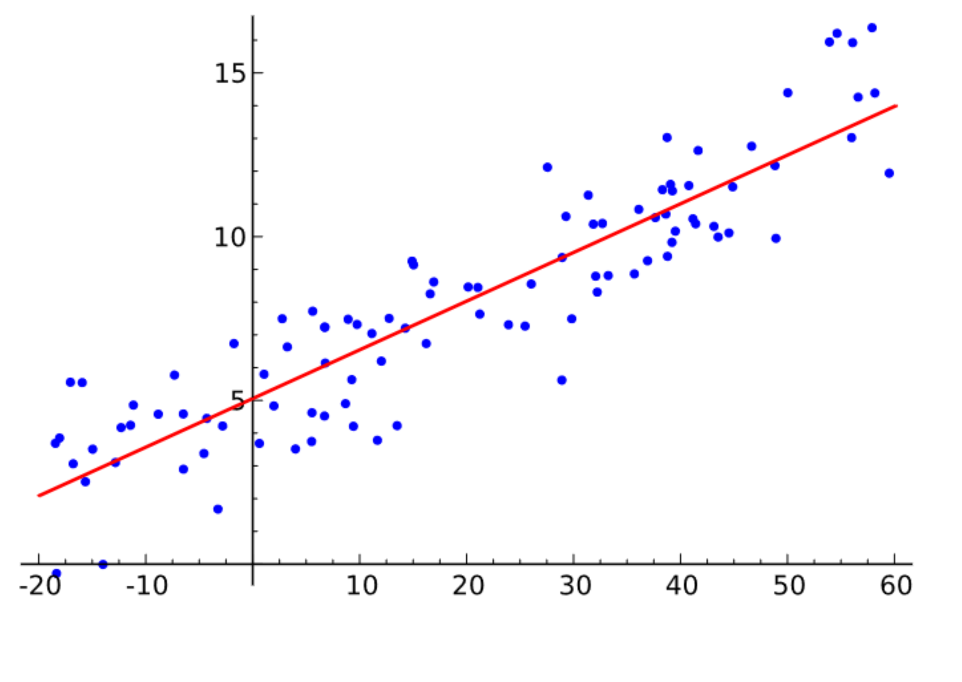
Non Linear Least Squares

Least squares problem

$$\min_x \|f(x)\|^2$$
$$f: IR^n \rightarrow IR^m$$



n number of unknonw, m nb of points
f non necessarily linear or differentiable

Linear regression (QUADRATIC)
Minimize distance between measurements ($n = 2$) and an unknown line ($m = 1$)

$$\frac{1}{2}|Qx - c|^2 = \frac{1}{2}(x^T Q^T Qx - 2x^T Q^T c + c^T c)$$

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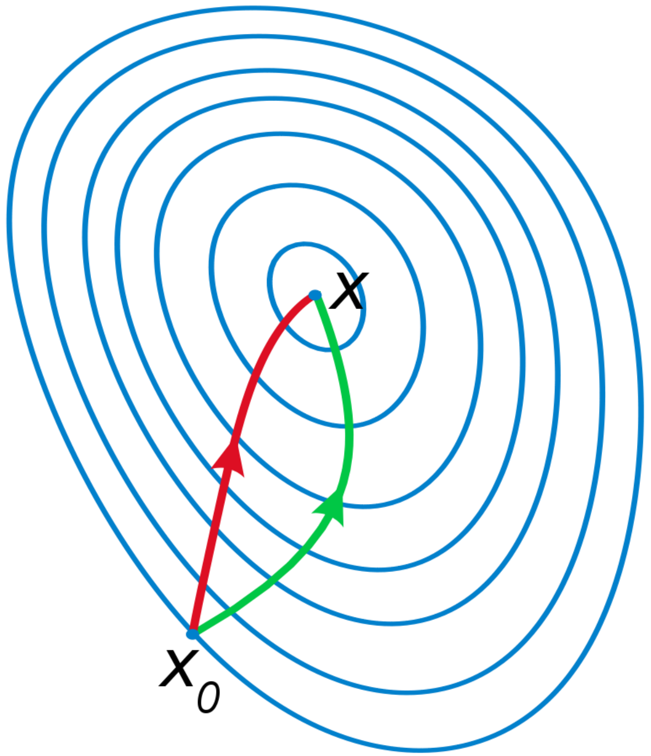
Non Linear Least Squares - lsqrsolve

Levenberg-Marquardt algorithm

To find a new direction:

- Gradient method (green) to provide steepest descent direction. Efficient when far from the optimal position.
- Gauss-Newton (red), approximate the problem as locally quadratic and solve it to find new position. Efficient when close to the optimal position.



Caracteristics

- Polynomial time

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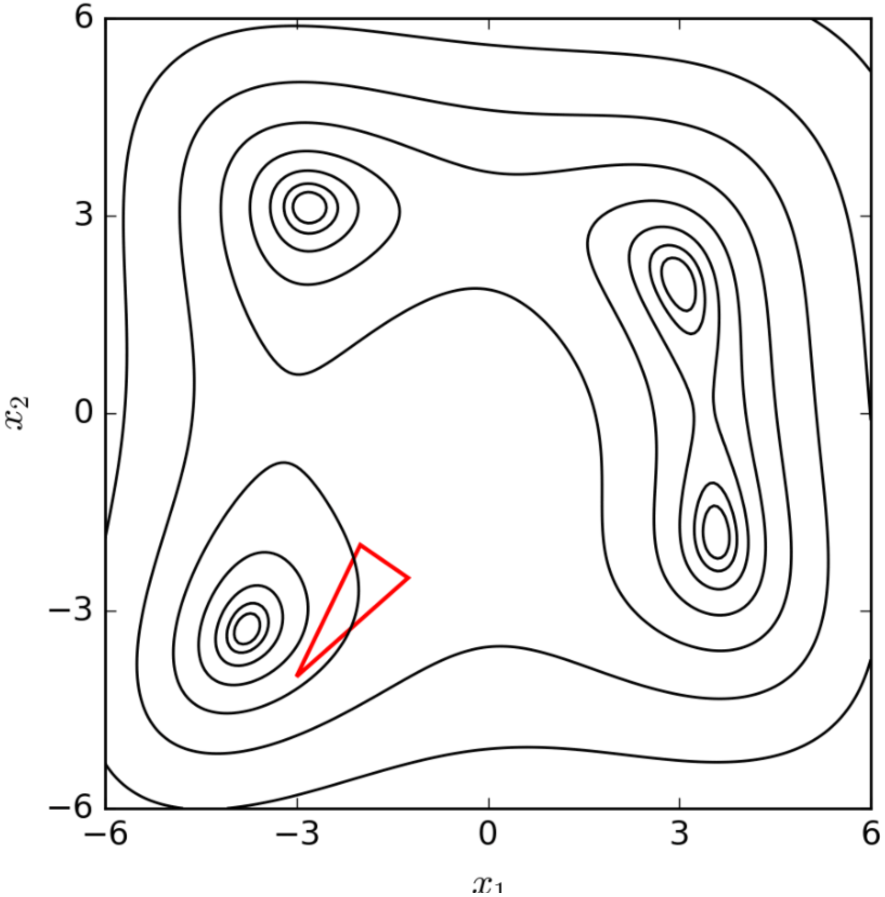
Non Linear optimization - fminsearch

Nelder-Mead Simplex

Simplex manipulation

3D Simplex



Caracteristics

- Non constrained
- High complexity
  Better suited for small problem
- Direct search

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Non Linear optimization - optim

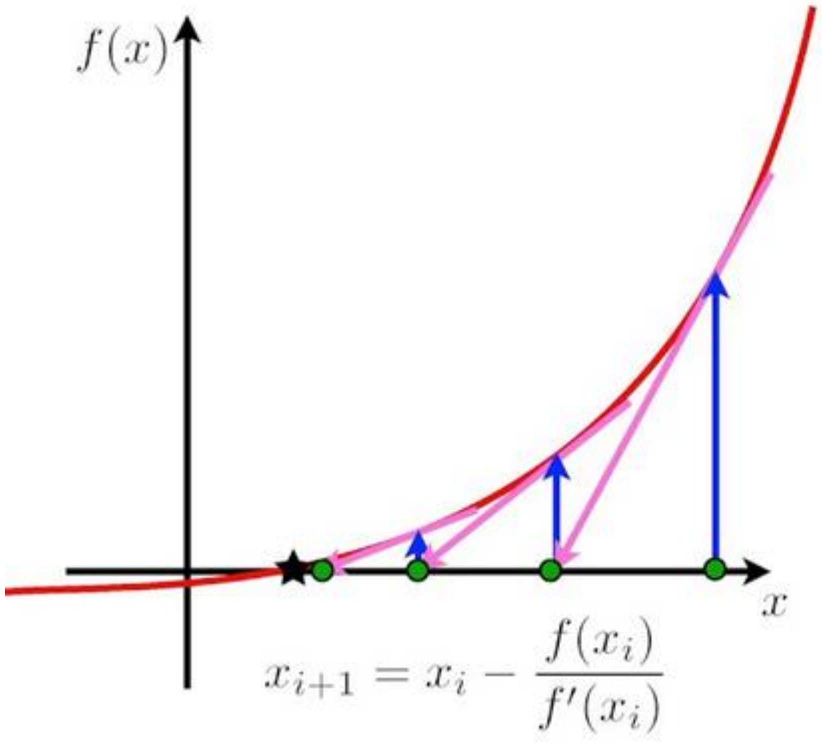
Quasi-Newton method

Quasi-Newton statement

$$x_{k+1} - x_k = B_k\big(f(x_{k+1}) - f(x_k)\big)$$

$$\Rightarrow x_{k+1} \approx x_k - \rho_k B_k f(x_k)$$

- Broyden-Fletcher-Goldfarb approximation for pour Hessian B_k
- ρ_k pour control convergence



Caracteristics

- High memory need to store B_k : O(n²) (O(n) in case of limited method)
- Gradient info must be provided
- Polinomial time O(n²) but not accurate

Nota

- Fsolve (alternative)
- Datafit / leastsq (built upon optim)

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Genetic Algorithms

GA Theory

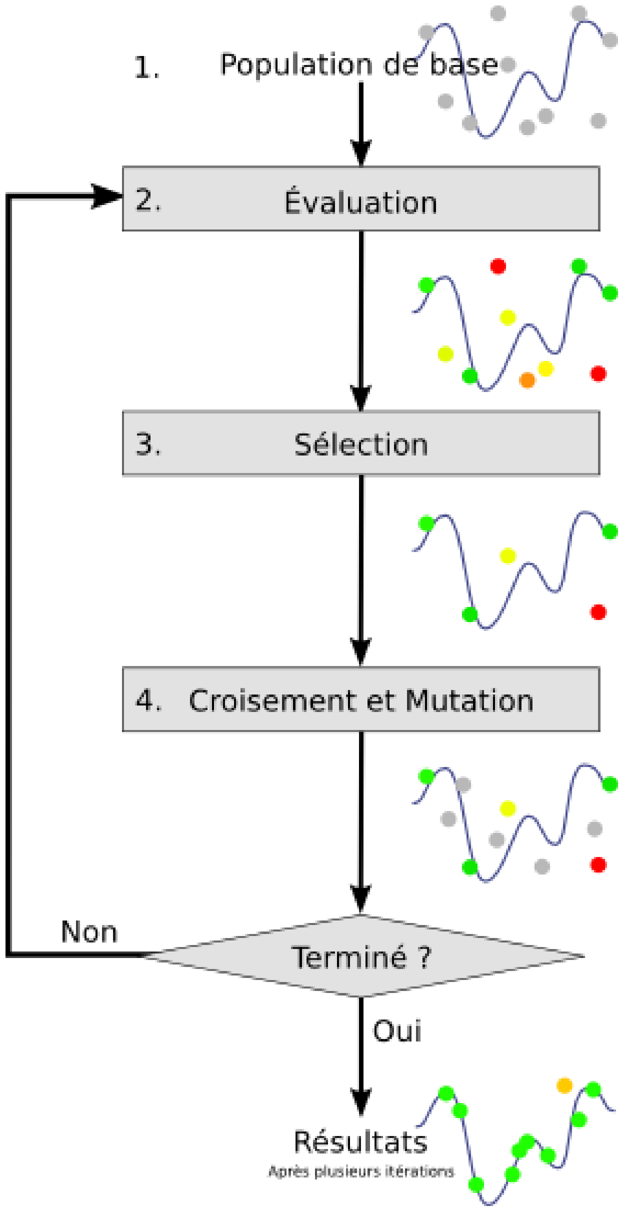
- Natural selection -
Probabilistics and non deterministics transitons:
Selection, Cross-over et Mutation

Caracteristics

- Bounds constraints
- Non Linear, Non Convex

Remarks

optim_ga /optim_moga: Single/Multi objective
pareto_filter: Non-dominated sorting



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Simulated Annealing

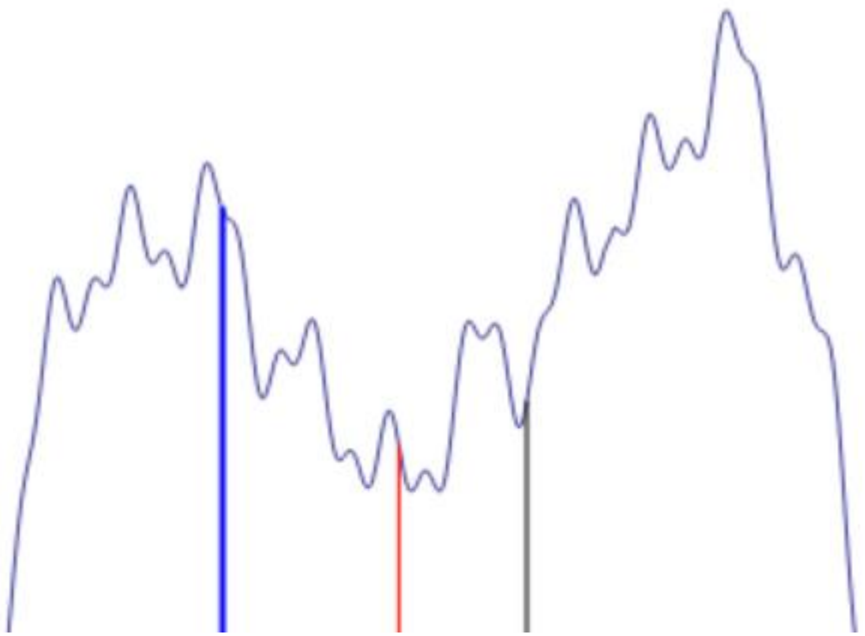
Simulated Annealing optim_sa

Empirical method based on metallurgical process

Alternate slow cooling cycles with heating cycles (annealing) in order to minimize material internal energy (strongest configuration)

Metropolis-Hastings Algorithm

Starting at a given state, we modify system towards another state. Either it get better (energy decrease) or it get worse.



Drawbacks:

- Empirical set-up
- Bounds constraints only

Benefits:

- Global optimum
- Discrete optimization

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